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The Liner Shipping Berth Scheduling Problem with Transit Times

Abstract

In this paper speed optimization problem of an existing liner shipping network is solved by adjusting the port berth times. The objective is to minimize fuel consumption while retaining the customer transit times including the transhipment times. To avoid too many changes to the time table, changes of port berth times are only accepted if they lead to savings above a threshold value. Since the fuel consumption of a vessel is a non-linear convex function of the speed, it is approximated by a piecewise linear function. The developed model is solved using exact methods in less than two minutes for large instances. Computational experiments on real-size liner shipping networks are presented showing that fuel savings in the magnitude of 2-10% can be obtained. The work has been carried out in collaboration with Maersk Line and the test instances are confirmed to be representative of real-life networks.

1. Introduction

Container shipping companies are currently facing combined challenges of overcapacity and volatile fuel prices. In addition, rising concerns about greenhouse gas emissions has made it crucial for shipping companies to reduce their fuel consumption. In the beginning of 2008, the future of maritime transportation looked remarkably bright. Major actors of the sector responded to an ever-increasing demand by extending the fleet capacity. At the end of 2008 orders for new ships were equivalent to almost 80% of the current fleet capacity [20]. However, when the economic crisis hit the liner shipping sector in 2009, a severe downturn in trade left the sector with overcapacity. As a direct consequence, freight rates dropped 28% on average [11]. As a response, shipping companies deployed less capacity on their networks and by the end of 2009, 12% of the global container fleet was laid up, compared to 3% one year earlier [11].

Another response to the overcapacity was slow steaming [5]. Slow steaming is reducing the speed on a service by increasing the overall service time. The slow steaming strategy has been employed by most container lines since 2009. While reducing bunker consumption, slow steaming will by definition also extend the round-trip time of a service. Since liner shipping companies generally provide weekly shipping services, the number of vessels deployed on a service would then increase with the duration of the round-trip. Because of this, more vessels are needed to operate the same tour and slow steaming can absorb some of the excess carrying capacity. This makes slow steaming the most relevant option to choose in order to reduce operational cost while utilizing the available vessel capacity. However, while slow steaming reduces the bunker consumption it may also extend the delivery times, resulting in unattractive service times for the customers. The delivery times can be defined as the duration for the transport of the demand from origin to destination and is in the remainder of this paper referred to as transit time.

According to Stopford [19], the bunker cost is 35% to 50% of a vessel's cost and according to [12] around 21% of the company expenses. Hence, bunker consumption is a critical item for achieving cost reduction. The 2013 maritime report of the United Nations [20] goes further by linking consumption cost and environmental concerns. As a result a better handling of fuel consumption may reduce both environmental impact and cost.

1.1. The Liner Shipping Berth Scheduling Problem with Transit Times

The slow steaming strategy exploits the relation between speed and bunker consumption. However, lowering the speed will obviously also result in longer transit times. Freight rates and transit times are crucial criteria for customers

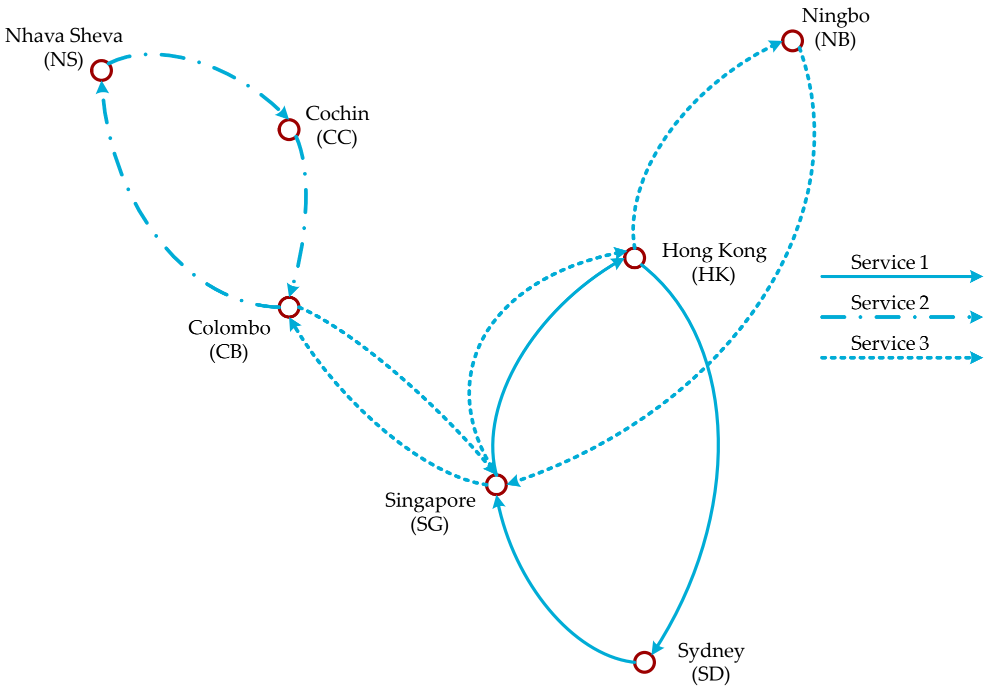


Figure 1: An example of a network containing three services with possible transhipment locations at Hong Kong, Singapore and Colombo. The network is similar to the one presented in [21].

when they choose a carrier. Brouer et al. [3] list nine parameters for the services that liner shipping companies can offer, mentioning that only freight rates and transit times are regarded as key factors. Consequently, the negative impact of slow steaming on the transit time could cause loss of customers. Therefore, lowering speed is a tradeoff between bunker consumption and customer satisfaction.

The networks of most liner shipping companies are organized around services that repeatedly serve a set of ports in a predefined sequence. A set of homogeneous vessels are deployed on the service to provide a periodic service, usually a weekly service. A service is defined by a port sequence, a timetable, a number of vessels deployed and a (weekly) frequency. In the example shown in Figure 1, port sequences of three constructed services are depicted. For clarity, the number of vessels and frequencies are omitted.

The implementation of slow steaming can be executed at two different stages of the network design process. It can either be implemented when the service is designed and then it will influence the port sequence and the number of vessels deployed. Alternatively it can be implemented when the service is already defined, so that only the schedule is re-optimized to smooth out the speed along the different parts of the service.

Both approaches have their advantages and drawbacks. The first method implies solving a large integrated problem. This may prove too complex to be solved by current techniques and implementing the solution may prove impossible for strategic reasons. The second method consists of optimizing subproblems individually as this is easier for a company to implement in their current network. This is, however, at the cost of possibly missing savings from a more holistic approach.

In the variant studied here, only the the arrival times in the serviced ports are rescheduled. We will denote it the *The Liner Shipping Berth Scheduling Problem with Transit Times* (LSBSPTT).

Earlier we defined a service as a port sequence, a timetable, a number of vessels deployed and a frequency. In the problem studied here the port sequence, the number of vessels deployed and the frequency will remain the same and only the timetable is changed. The arrival times define the schedule for each service of the network and they are limited by the sailing speed. This is why these arrival times are the decisions variables of the LSBSPTT. The slower a vessel sails the lower the bunker consumption, and therefore the arrival time will be as late as possible when minimizing bunker consumption. The fleet size and the vessel mix, the round-trip time of services, the port sequence of services and the frequency of a service are all fixed so that the only change to be implemented is modified times for the port calls.

With the number of vessels fixed and a weekly frequency of port visits, the overall duration of a round trip is established. Weekly frequency is the standard for most liner shipping operations. The duration of a round trip (measured in weeks) is therefore equal to the number of vessels assigned to the service. Since we do not want to change the number of vessels of a service, the overall round-trip time of a service is respected in the LSBSPTT.

In order to calculate transit times, all port visit times are calculated from a given time zero. The first port call on a service is defined as the starting port. In order to allow the model to also change the port visit time of the starting port, the choice of starting port is a variable in the model.

At most ports in Europe, North America and Asia a liner shipping company needs to book in advance a berthing time for a port call with the port authorities. Changing a booked berthing time creates additional administrative work, and the port authorities may not always be able to accommodate the requested change. Therefore it is important that berth times are only changed if they contribute to significant savings. Hence, a penalty is introduced for rescheduling a port visit. This penalty is independent of how much the time is changed, since the administrative expenses are considered to be the same.

Liner shipping offer the transportation of customer cargo from a port of origin O to a port of destination D. A path linking O and D is called a routing. Linking two ports can be done by a direct routing where the vessel leaving port O is the same as the one reaching port D. It can also be completed by using transhipment routing where the cargo is transhipped from one vessel to another vessel at a port different from O and D. A transhipment routing can easily include several of these transhipment operations. Transhipments generally add flexibility to a liner shipping network and ensure good capacity use.

There may exist several different routings for a single demand (from its origin to its destination), as one routing may not have enough capacity for all the cargo. Finding the cheapest and/or fastest routings satisfying the vessel capacities is a variant of the multi commodity flow problem. In the case of the LSBSPTT, the routing problem has already been solved beforehand. With the cargo routing already defined, each of the routes have already been selected and now the difficulty is to ensure satisfaction of the time restriction on cargo for reaching their destination. As mentioned earlier the customers are mainly interested in the price and the duration of the transport. The transit time of a container routing includes both the time spent on a vessel and time spent waiting at a port for a vessel during a transhipment. This waiting time is here referred to as the transhipment time. The transit time is a fixed parameter for each origin destination pair and is important for staying competitive on the market, hence, respecting it is a constraint in the LSBSPTT.

The model should make sure that all transit times for the cargo are retained. If a container is transhipped in a port from a service A to a service B some minimum time between the berthing of the two vessels is needed. The minimum time is called the *Connecting Time Window* (CTW) and it is usually measured from the departure of vessel A to the arrival of vessel B. Most ports operate with a CTW of eight hours, but the model allows individual settings for each port. If the time between vessels A and B is below the CTW, or vessel A departs after the arrival of vessel B the given container has to wait for the next weekly arrival of vessel B.

In some specific situations a *hot berthing* can take place if the berth locations of the two vessels A and B are close to each other. In this case a container is loaded directly from vessel A to a truck that drives to vessel B. In this case less than eight hours are needed as CTW. The presented model supports any value of CTW and one can, if necessary,

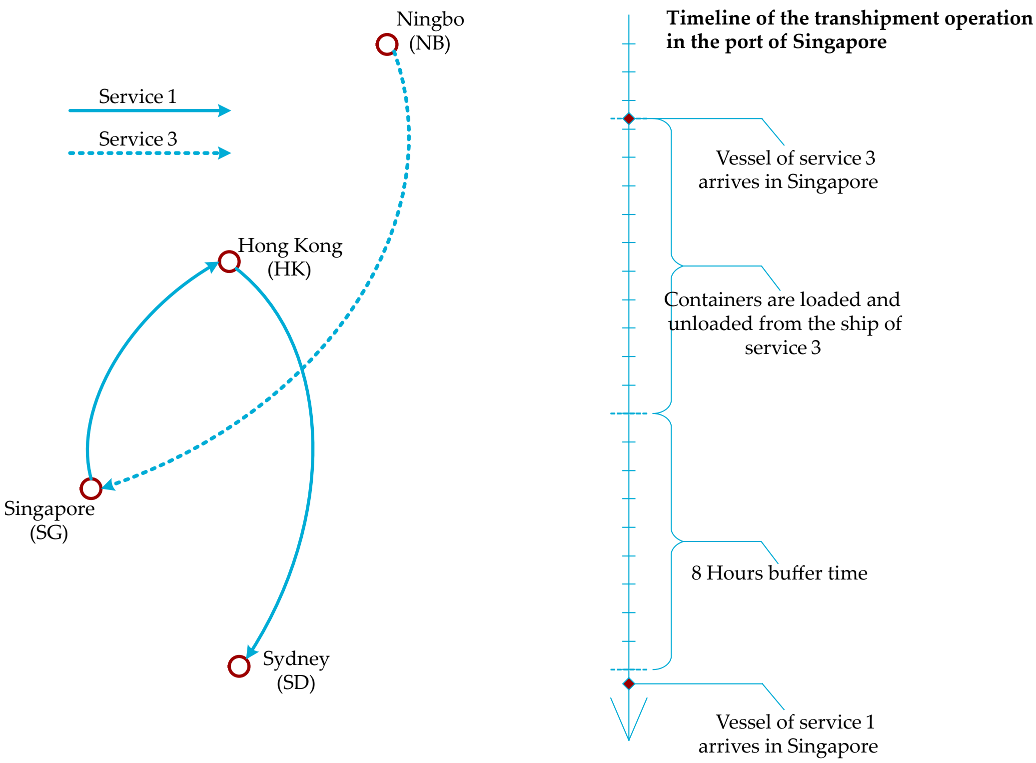


Figure 2: An example of a routing from Ningbo to Sydney on the network from Figure 1. The routing shown contains a transsshipment operation in Singapore which is described to the right.

keep some of the berthing times fixed to ensure that a specific hot berthing will be possible.

As mentioned earlier, the fleet size, vessel mix, round-trip time of services, port sequence of services and the frequency of a service are all fixed parameters for the LSBSPTT.

To summarize, the considered problem aims at minimizing the fuel consumption of a liner shipping network by rescheduling port call times while retaining the sequence of port calls and retaining customer transit times. To the best of our knowledge, this problem has not been considered before in the literature. Slow steaming, as implemented by most liner shipping companies, has a fast headhaul trip and a slower backhaul trip. The here proposed solution tries to smoothen the speed in the whole round trip within the limits given by customer transit times. The proposed solutions are easy to implement for a liner shipping company, since no changes to the logic of the network are made.

The remaining part of this paper is structured as follows: A thorough literature review is presented in Section 2. The review focuses on research sharing properties with the LSBSPTT. In Section 3 the constraints and variables of the problem are described together with a detailed description on the methodology used to handle the non-linearity of the problem. In section 4 the entire Mixed Integer Programming model (MIP) is presented. In Section 5 the different test instances are described which is followed by computational results in Section 6. Finally, in Section 7, the observations are discussed, and future improvements are considered.

2. Literature review

Until the new millennium, liner shipping transportation was scarcely studied in the area of operations research (see [7]), as opposed to truck transportation, airline transportation and train transportation.

Literature linked to maritime optimization problems and liner shipping network design is reviewed in [6], [7], [8], [18] and [24]. Brouer et al. [3] presented in 2014 a benchmark suite to help the research development providing a good overview of the concepts of liner shipping and how the liner shipping network is structured.

Bunker cost accounts for a large share of the total liner shipping cost: 35-50% ([15], [18] and [19]); hence, reducing bunker consumption can result in a considerable reduction in the cost. Plum et al. in [16] give an introduction to optimization techniques for modelling bunker purchasing and how this may lower the bunker cost for a liner service.

Stopford [19] explains that solving scheduling problems can bring what is called *economics of speed* through a decrease in bunker consumption. Cariou in [5] assesses the environmental impact of slow steaming and conclude that it is also a mean to reduce the carbon footprint of the liner shipping companies; thus adding an environmental incentive on top of the economic incentive. This means that the companies can decrease their cost and at the same time reduce their carbon emission resulting in a win-win situation.

When proceeding to slow steaming optimization, it is important to know how the consumption grows as a function of speed. There has been different suggestions for how to calculate consumption from the speed. Brown et al. [4] in

Reference	Considers: Transhipment	Round trip	Transit time	Consumption function	Problem type	Solution method	Instance size
Brown et al. 1987 [4]	No	No	No	Super linear function	Tramp shipping	Column generation	Small.: 12 ports and 50 cargos
Fagerholt et al. 2010 [9]	No	Time windows	No	Quadratic function	Tramp shipping	Shortest path problem	Small: 16 ports and no cargo
Norstad et al. 2011 [14]	No	Time windows	No	Quadratic function	Tramp shipping	Heuristic method	Medium: 40 ports and 70 cargos
Meng and Wang 2011 [13]	No	Yes	Yes	Cubic function	Liner, Only long haul	Outer approximation for non linearity	Small: 12 ports, and 1 service
Reinhardt and Pisinger 2011 [17]	No	Yes	No	Fixed Speed	Liner, Network Design	Cutting Plane	Small: 15 ports
Wang and Meng 2012 [22]	Yes	Yes	No	Unique law per leg	Liner, Cargo routing included to minimize transit time	Outer approximation for non linearity	Medium: 46 ports, 11 services
Wang and Meng 2012 [21]	Yes	Fixed start time	Yes	Cubic function	Liner, Probabilistic version of speed with fleet deployment	Cutting plane based method piecewise linear	Medium: 46 ports, 11 services and 100 demands
Wang et al. 2013 [23]	No	Yes	Yes	Unique law per leg	Liner, Speed, fleet deployment	Dynamic Programming and other methods	Small: 7 ports, 1 service
Karsten et al. 2013 [10]	Yes	Yes	Yes	Not included	Liner, Flow	Column generation	Large: 111 ports 4000 demands
This paper	Yes	Yes	Yes	Cubic function	Liner, Speed, Rescheduling penalized	branch and bound Linearization of cubic function	Large: 226 ports, 300+ services, 10000+ demands

1987 suggested the super linear function based on generated schedules, while Notteboom [15] in 2009 suggested an empirical consumption function extrapolated from published data. Nevertheless, engine theory and empirical data are most commonly linking sailing speed and bunker consumption through a cubic function approximation (Alderton [2] and Stopford [19]). In an empirical study Wang and Meng [22] confirm that the cubic function is a good approximation for the conversion from the speed to the consumption. Clearly the cubic function is nonlinear which can complicate linear models and therefore speed optimization is often not considered when optimizing networks ([1], [3] and [17]).

In Table 2 we give an overview of literature in maritime shipping which considers the bunker consumption in their cost optimization. The first column lists in chronological order articles considering bunker consumption. The transhipment column states whether transhipment operations are allowed. The round trip column indicates if limits on the total round trip time of services is considered in the model. The transit time column shows whether there is a time restriction on the journey of the container from origin to destination. In the fifth column the consumption law used is provided. The sixth column gives an indication of the problem solved. All the models considered except for [21] and this paper do not consider the time waiting at port during a transhipment.

From Table 2 it can be observed that most of the solution methods have only been tried on small or medium instances. This implies that none of the cited works had to face large-scale problems, in the magnitude of the problems world leading liner shipping companies must deal with.

Wang and Meng [21] formulate a time scheduling problem using a probabilistic interpretation of the arrival time, which result in a probabilistic version of the model. In [21] the existing berth times are not considered. This would be applicable if the company is generating completely new services with no previous calls to the ports on the service. In [21] it is not possible to change the temporal starting point for the services and it introduces its piecewise linear function by applying a cutting plane algorithm to solve a medium sized instance with 100 demands. Keeping a fixed temporal starting point restricts the possibilities for changing the time for the port visit during the first and last week and therefore the model may not always provide the minimal solution in cases where there is less than a week of travel time between the last and the first port.

In this paper we present a new version of the speed optimization problem where existing port reservations are rescheduled. This is an incremental result expanding the previous work presented by Wang and Meng [21] and on larger problem instances. In the problem presented in this paper changing berth time reservations is penalized to ensure that changes do not occur without ensuring a significant reduction in bunker consumption. Since we can compare to the existing schedule we will be able to show the reduction in bunker consumption achieved by the rescheduling. The problem model also allows for locking the berth time on some of the ports on one or more services. The required connection time needed for transhipment between services can be changed depending on the services involved in the transhipment. We present a model for this new problem of rescheduling existing port visits to minimize bunker consumption under a rescheduling penalty. This model also includes a variable starting point for the service to ensure that cases where there can be less than a week between two port visits are solved to optimality. The problem is solved for very large instances by using the fact that the cubic function is convex and therefore we can represent it by a series of linear constraints. However, we do not route the containers. In real life finding the optimal flow on a network is complicated due to various rules such as cabotage, sanctions and others. The companies generate an optimal default flow set on a network when the network is altered. Changes in routing of demands are often only carried out for a few demands as the changes may create implementation chaos in the operation with containers ending up on the wrong vessels or sanctions and cabotage rules may be violated. Thus we use the routes already implemented. The solution found by our algorithm is as a result easier for the company to implement as it only involves changing port visit times.

By using an network resembling a real-life network we can get a somewhat realistic estimate for the amount of reduction in bunker consumption that can be achieved by a company by simply changing the time of the port calls. As we shall see later the number of routes with constrained transit times and containing transhipment is what increases the complexity of the problem.

3. Mathematical Problem formulation

In this section the LSBSPTT is formulated more formally. The objective function and the various constraints will be described in mathematical terms. Notation will be introduced when needed, but Table 1 gives an overview of all variables and parameters used in the model.

Let R be the set of all services, each service having a duration which is a multiple of a week. Let S_r be the number of weeks representing the duration of a service $r \in R$. Let L_r be the ordered set of legs on service $r \in R$ where we use the terminology $(l', l) \in L_r$ to indicate that leg l' is followed by leg l in the port visit sequence. Note that the leg sequence is cyclic so that the last leg in L_r is followed by the first leg. Then, let t_l^d be a continuous nonnegative variable stating the departure time from the end port of leg $l \in L_r$, and let t_l^a be a continuous nonnegative variable stating the arrival time at the end port of leg $l \in L_r$. The domain for t_l^d and t_l^a is $[0, 168S_r[$ where 168 is the number of hours in a week. The time unit used for all parameters and variables in the model is hour.

3.1. Objective minimizing bunker cost

The overall aim of the model is to minimize the bunker consumption using penalties to restrict the modified port visit times to those generating significant savings. Using the currently reserved port visit time is preferable since the time slot is available in the port, and customers are used to this time. We will therefore not change the port visit time unless the savings are somewhat significant. Therefore we introduce a penalty for each changed port visit time.

3.2. Scheduling port visits

All services use the same global time (Greenwich Mean Time) starting from Sunday at 24:00. As seen in Figure 1 all services are cyclic. Every service will have exactly one first port visit on the route, defined as the first port called after time zero. A binary variable f_l is used to indicate if leg l ends at the first port visit of service r. Ensuring that exactly one start leg is selected is modeled by the following constraint:

$$\sum_{l \in L_r} f_l = 1, \quad r \in R \tag{1}$$

The time used on a leg l is the time the vessel departs from the end port of leg l, t_l^d, minus the time the vessel depart from the end port on the previous leg $t_{l'}^d$ where $(l', l) \in L_r$. When l is the first leg on the service we must furthermore add the service time $168 \cdot S_r$ to get the time used.

3.2.1. Portstay and pilottime

The time used on a leg $l \in L$, which can also be described as the time between $t_{l'}^d$ and t_l^d, where $(l', l) \in L_r$, can be separated into different tasks. Such tasks are here listed in the order they occur between time $t_{l'}^d$ and t_l^d :

Pilot out This is the pilot time used on leaving the start port of the leg.

Ocean sailing time This is the time the vessel sails without a pilot. The speed during this segment is determined by the liner operators and is the only time which is adjustable.

Pilot in This is the pilot time used on entering the end port of the leg.

Portstay This is the time the vessel spends at berth in the port to unload and load containers, load bunker and other service tasks. For a leg l the portstay at the end port is included in the departure time t_l^d of leg $l \in L$. The portstay at the destination port of leg l is denoted as H_l^{stay}

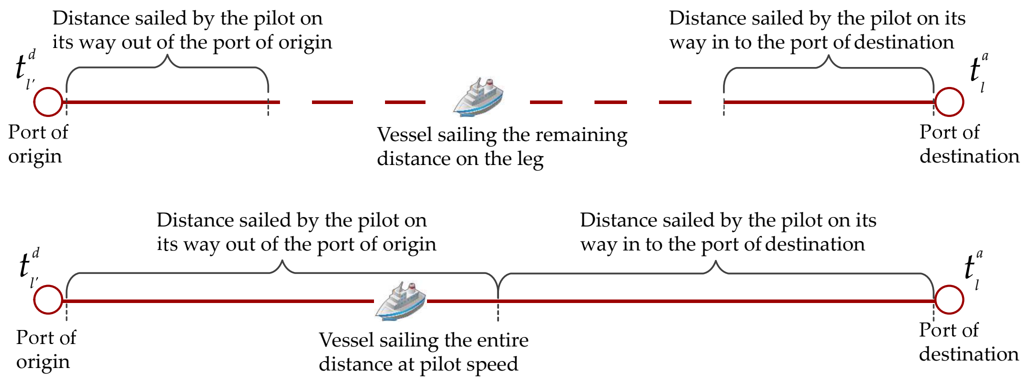


Figure 3: A graphic representation of the different parts of a leg. The top diagram shows the case where there is some distance on the leg that is not part of the piloting. The diagram below shows the case where the leg distance is equal to the pilot out and pilot in required on the leg.

Let the time on a leg used on piloting (both out and in) be denoted P_l^{pilot}. When piloting a fixed speed is used and therefore every leg l has an associated distance D_l, excluding the piloting, where speed optimization can take place. For ports located very close to each other D_l may be zero, leaving the leg with no room for speed optimization. For a graphic illustration of the piloting on legs see Figure 3.

Let the variable τ_l be the time used on sailing the distance D_l on leg l. The variable τ_l can then be defined as:

$$\tau_l = t_l^d - t_{l'}^d - H_l^{stay} - P_l^{pilot} + 168 S_r f_l, \quad (l', l) \in L_r \tag{2}$$

Note that by definition τ_l will always be positive. In fact τ_l will be greater than or equal to the time it takes to sail the distance D_l at maximum speed. Clearly the speed used for sailing leg $l \in L$ can be derived from the time τ_l and distance D_l. In the model the speed is modeled using time and distance.

3.3. Maximum and minimum sailing speed

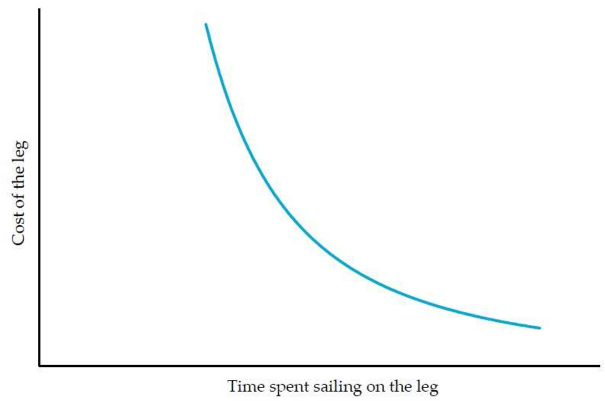
Let T_l^{min} be the time needed for sailing the distance D_l at maximum speed and let T_l^{max} be the time needed for sailing D_l at the minimum sailing speed. Clearly, the time spent on sailing a leg l must be greater than or equal to the time needed to sail the distance at maximum speed, so we have $\tau_l \geq T_l^{min}$. The vessels also has a minimum speed which is required to be able to maneuver the vessel. This minimum speed is only used for calculating the bunker consumption. The minimum speed does not restrict the time spent on the leg which can be much larger allowing for the ship to lay waiting for their berth time outside a port.

Let $C_l(\tau_l)$ be the cost of the bunker used when sailing the leg l using time τ_l. Then we have the constraint that $C_l(\tau_l) \geq C_l(T_l^{max})$. In other words the minimum speed will provide a lower bound for the amount of bunker required for sailing a leg. The different ports have different piloting distances. The distances depend on how accessible the port is from the sea. However the pilot speed and time is assumed to always be the same and therefore it can be excluded from the optimization.

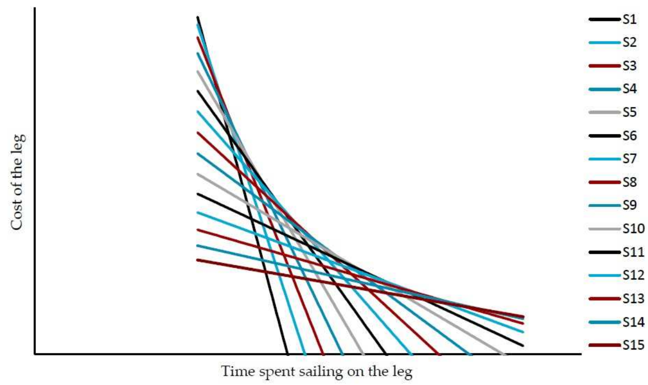
3.4. Bunker consumption

For the bunker consumption we use the cubic function introduced by [3]:

$$B(\delta) = \left(\frac{\delta}{\delta_v}\right)^3 B(\delta_v), \tag{3}$$



(a)



(b)

Figure 4: Figure (a) shows the bunker consumption on a leg as a function of time used on sailing the leg. Figure (b) The bunker consumption on a leg approximated by 15 secants, denoted S1-S15.

where δ is the speed used and δ_v is the design speed of vessel class v. $B(\delta_v)$ is the bunker consumption at the design speed and $B(\delta)$ is the bunker consumption at speed δ. The design speed δ_v and the bunker consumption at design speed $B(\delta_v)$ are part of the specifications of a vessel and therefore known for each vessel and engine.

To calculate the bunker consumption cost we will apply the bunker price using the duration of the leg and the given distance of the leg. The time used when sailing the leg l at design speed of vessel type v is denoted as τ_v. Thus the speed can be found as $\delta_l = \frac{D_l}{\tau_l}$ and $\delta_v = \frac{D_l}{\tau_v}$ where δ_l is the speed used on leg l. By using the conversion from speed to time, equation (3) can be reformulated as:

$$\hat{C}_l(\tau_l) = \left(\frac{\tau_v}{\tau_l}\right)^3 B(\delta_v) C_T, \tag{4}$$

where C_T is the price per metric tonne of bunker and $\hat{C}_l(\tau_l)$ is the price for bunker consumed at every time unit when using the time τ_l to sail the distance D_l on leg l. The time unit is hour when speed is nautical miles per hour. Multiplying both sides of equation (4) with τ_l one finds the bunker cost used on sailing D_l using time τ_l. Thus getting:

$$C_l(\tau_l) = \left(\frac{\tau_v}{\tau_l}\right)^3 B(\delta_v) C_T \tau_l. \tag{5}$$

The function in equation (5) is illustrated in Figure 4 (a). To circumvent the nonlinearity of this function we have chosen to make a piecewise linear representation. Since the bunker cost function is convex we can construct a piecewise linear representation by using a set of linear constraints. We have chosen to do this by a set of secants to the function. These secants are evenly distributed along the curve as illustrated in Figure 4(b). The number of secants used in the approximation is provided as input.

Let P be the set of secants used to approximate the bunker cost. The linear functions of the secants must be generated for each leg to account for the varying distance. For each secant a linear function of $\hat{C}_l(\tau_l)$ is defined as:

$$\hat{C}_l(\tau_l) = \phi_l^p \tau_l + \omega_l^p, \tag{6}$$

where ϕ_l^p is the slope of the secant $p \in P_l$ of the leg l and ω_l^p is the secants intersection with the y axis. The constraint used for ensuring that the cost of sailing a leg l satisfies the secant approximation can then be formulated as:

$$\hat{C}_l = \phi_l^p \tau_l + \omega_l^p, \quad p \in P, l \in L, \tag{7}$$

where $\hat{C}_l$ is the cost of sailing leg l at time τ_l.

3.5. Demands and their transit time constraints

Let Q be the set of demands which are to be transported by the shipping company. Each demand $q \in Q$ contains an origin, destination, amount of containers, duration of transport and a price. The company has a set of routes used for shipping the demand. The reason for using the existing set of demand routes is that the company needs to know that there is capacity for the amounts and that restrictions such as cabotage and embargoes, port visit draft and so forth are satisfied as explained in the introduction.

When rescheduling the port visits the transit time of a demand may change. It is important that the company can make sure that the transit time stays within their requirements so that customers are not lost. The transit time of a demand $q \in Q$ is denoted as TT_q.

The transit time can be the time the container is onboard the vessels, which can be calculated from the time t_l^d of the legs on the route. If the route contains transhipments then the time the container must wait at a terminal for the next vessel must also be added to the transit time.

The services have a weekly departure from each port; therefore a container will not wait in a port longer than a week plus the time required for the connection between the vessels. The required minimal connection time however can be from a few hours up to several weeks. Thus the time of week the port visit occurs must be determined. To calculate the time of week the port visit occurs, an integer variable w_l is used. The variable $w_l^d = \left\lfloor \frac{t_l^d}{168} \right\rfloor$ represents the whole number of weeks completed at time t_l^d.

Let C be the set of connections $c = (l, h)$ where $l \in L_r$ ends at a port i and leg $h \in L_s$ starts at port i and service $r \neq s$. Let C_q be the connections used by demand $q \in D$. Note that two different demands q and q' may have connections in common. Let CT_c^{min} be the minimum required connection time for the connection c. Moreover, let CT_c^{weeks} be the number of weeks of CT_c^{min} so that $CT_c^{weeks} = \left\lfloor \frac{CT_c^{min}}{168} \right\rfloor$.

Let the variable $t_l^a = t_l^d - H_l^{stay}$ be the arrival time at the end port of leg l. Then let w_l^a be an integer variable so that $w_l^a = \left\lfloor \frac{t_l^a}{168} \right\rfloor$ represents the whole number of weeks completed at time t_l^a.

The time the container must wait in port at a connection going from a service with leg l to another service on leg h can be expressed as two different cases: The first case is the case where a demand using a connection where the arrival time of the vessel on which the container must leave is later in the week than CT_c^{min} after the departure of the vessel on which the container arrives. In this case the waiting time can be expressed as:

$$t_h^a - t_l^d - 168(w_h^a - w_l^d - CT_c^{weeks}) \geq CT_c^{min} \tag{8}$$

The second case is the case where the connection is less than $CT_c^{min} - CT_c^{weeks}$ then the container must wait an additional week at port for the next arriving vessel. To model these two cases we include the variable x_c which represents the number of weeks to be added when using connection $c \in C$.

Using this we formulate the connection requirements with the following constraint:

$$t_h^a - t_l^d - CT_c^{min} - 168(w_h^a - w_l^d - CT_c^{weeks} - x_c) \geq 0, \quad c = (l, h) \in C \tag{9}$$

The constraints can now be used to formulate the transit time constraints. As mentioned earlier the transit time consists of the time spend on the legs and the time spend in the terminal during a transhipment also called the connection time.

$$\sum_{c=(l,h)\in C_q} (t_h^a - t_l^d - 168(w_h^a - w_l^d - x_c + CT_c^{weeks})) + H_h^{stay} + \sum_{l\in L_q} (t_l^d - t_{l'}^d + 168 S_r f_l) - \hat{H}_q^{stay} \leq TT_q, \quad q \in Q \quad (10)$$

In this constraint TT_q is the transit time limit of demand q and the $\hat{H}_q^{stay}$ is the port stay at the destination port of demand q. The set L_q contains the legs on which the cargo is sailed. These constraints are added to the model for all demands transported on legs where speed may be changed.

4. Complete Model

type	notation	description
sets	R	set of services
	L	set of legs where $l \in L_r$ is a leg on service $r \in R$.
	Q	set of demands defined by a route (as set of legs $l \in L$) between to ports A to B
	C	set of connections between service $l \in L$ and $\hat{l} \in L$ used by the demands in Q
	P_l	set of secants used for approximating the bunker curve on leg $l \in L$
parameters	$\hat{P}_l$	penalty for shifting a berth time at leg l
	S_r	number of weeks used for the round trip of a service
	H_l^{stay}	the portstay of leg $l \in L$
	$\hat{H}_q^{stay}$	the portstay of the last leg of demand $q \in Q$
	P_l^{pilot}	time used for piloting on leg $l \in L$
	D_l	distance of leg $l \in L$
	T_l	current scheduled time for the berth visit
	T_l^{min}	minimum time used for sailing leg $l \in L$ (at maximum speed)
	TT_q	transit time requirement of demand $q \in Q$
	CT_c^{min}	minimum time required time for connection $c \in C$
	CT_c^{weeks}	equivalent to $CT_c^{min} \mod 168$ for connection $c \in C$
	ϕ_l^p	gradient of secant $p \in P$ of leg $l \in L$
	ω_l^p	y-axis intersection of secant $p \in P$ of leg $l \in L$
variables	f_l	(binary) indicates if leg l is a start leg of a service
	t_l^d	(continuous) departure time of leg $l \in L$ at its end port
	t_l^a	(continuous) arrival time of leg $l \in L$ at its end port
	C_l	(continuous) cost of of sailing leg $l \in L$
	w_l^d	(integer) number of weeks from start of leg $l \in L$. This means that $w_l^d = \left\lfloor \frac{t_l^d}{168} \right\rfloor$.
	w_l^a	(integer) number of weeks from start of leg $l \in L$. This means that $w_l^a = \left\lfloor \frac{t_l^a}{168} \right\rfloor$.
	m_l	(binary) is one if the port visit time has been changed
	x_c	(integer) indicates the number of weeks needed to make the connection

Table 1: Overview of notation used in the model

In the previous section some of the different components of the model were explained. In this section we present the complete model. An overview of the notation can be seen in Table 1. Moreover we use the notation $(l', l) \in L$ to indicate that l' is the previous leg of l.

$$\min \sum_{r \in R} \sum_{l \in L_r} C_l + \hat{P}_l m_l \tag{11}$$

$$\sum_{l \in L_r} f_l = 1, \qquad r \in R \tag{12}$$

$$t_l^d - t_l^a = H_l^{stay}, \qquad l \in L \tag{13}$$

$$t_l^a - t_{l'}^d - P_l^{pilot} + 168 S_r f_l \geq T_l^{min}, \qquad r \in R,\ (l', l) \in L_r \tag{14}$$

$$T_l - (t_l^d - 168(w_l^d) + 168 m_l) \geq 0, \qquad r \in R,\ l \in L_r \tag{15}$$

$$t_l^d - 168 w_l^d - T_l + 168 m_l \geq 0, \qquad r \in R, \quad l \in L_r \tag{16}$$

$$\phi_l^p (t_l^a - t_{l'}^d - P_l^{pilot} + 168 S_r f_l) + \omega_l^p \leq C_l, \qquad r \in R,\ (l', l) \in L_r,\ p \in P_l \tag{17}$$

$$0 \leq t_l^d - 168 w_l^d \leq 168 - \epsilon, \qquad r \in R,\ l \in L_r \tag{18}$$

$$0 \leq t_l^a - 168 w_l^a \leq 168 - \epsilon, \qquad r \in R,\ l \in L_r \tag{19}$$

$$t_h^a - t_l^d - CT_c^{min} - 168(w_h^a - w_l^d - CT_c^{weeks} - x_c) \geq 0, \qquad c = (l, h) \in C \tag{20}$$

$$\sum_{c=(l,h) \in C_q} (t_h^a - t_l^d - 168(w_h^a - w_l^d - x_c - CT_c^{weeks}) + H_h^{stay} + \sum_{(l,l') \in L_q} (t_l^d - t_{l'}^d + 168 S_r f_l) - \hat{H}_q^{stay} \leq TT_q, \qquad q \in Q \tag{21}$$

$$w_l \in \{0, ..., S_r - 1\}, \qquad r \in R, l \in L_r \tag{22}$$

$$t_l^a, t_l^d \geq 0, \ \ c_l \geq 0, \qquad r \in R, l \in L_r \tag{23}$$

$$f_l, m_l \in \{0, 1\}, \qquad r \in R, l \in L_r \tag{24}$$

$$x_c \in \{0, 1, 2\}, \qquad c(l, \hat{l}) \in C \tag{25}$$

The objective (11) minimizes the sum of bunker cost and penalties for moving port call times. The parameter $\hat{P}$ is the penalty for moving the port call time at the end of leg $l \in L$, and the binary variable m_l is one iff the port call time has been moved. The variable C_l is the bunker cost on leg l.

The first constraints in the model (12) ensure that exactly one leg is chosen as the first for each service. The arrival time of leg l is defined in equation (13) Constraints (14) ensure that the legs are not traversed at a faster speed than the maximum speed of the vessel. Constraints (15) and (16) ensure that a penalty is applied if port time is changed from the original scheduled (weekly) port visit time T_l for leg $l \in L$. For every leg the consumption is restricted by a set of linear functions represented by constraints (17). In constraints (17) the variable ϕ_l^p is the slope of secant p_l on leg $l \in L$ and ω_l^p is the intersection of the secant. Constraints (18) and (19) define the value of w_l^d and w_l^a respectively. Constraints (20) ensure that the container waits at the terminal for the next vessel arriving after the minimum required connection time CT_c^{min}. The transit time is ensured to be below the requirement TT_q for demand $q \in Q$ with constraint (21). Constraints (22) to (25) define the variable domains. The variables (23) are continuous variables indicating the time and cost.

5. Test data

To test the algorithm a network containing 308 services is used. The services are services existing in the world operation today.The services in the network have a weekly frequency and port visit times are applied to all ports. These port visit times are taken from existing services published on the Internet by the various companies.The demands are constructed with help from a liner shipping company. For large liner shipping companies the number of distinct demands is around 20,000. Note that each demand is defined by an origin, destination and path. This path is unique for each demand. Therefore several demands with the same origin and destination may exist.

network properties	
services	308
legs	2091
demands	20863
connections	4722
total routes	20863

Table 2: Various properties of the constructed network

instance	services	port visits	demands	connections	legs
Single-service	1	21	1403	273	21
Two-services	2	38	2459	482	38
11-services	11	237	8435	1745	237
Cluster-1	28	137	3130	418	151
Cluster-2	29	72	4038	388	97
One-operator	84	1062	20863	4576	1062
Whole-network	308	2091	20863	4722	2091

Table 3: Various properties of the considered instances

The same network is basis for all tests. The characteristics of the network are listed in Table 2. Based on this network we have created a number of test instances. These test instances are created by selecting some subset of the ports called and allowing the visits to be rescheduled. In the instance named *Single-service* we open all port visits on a single service, which consists of 21 port visits and 273 demands uses this service as part of their journey from origin to destination. In the *Two-services* instance all port visits on two services are opened. The two services contain the single service used in the *Single-service* instance. The two services chosen call some of the same ports and there exists demand which tranships between the two services. A larger instance named *11-services* contains 11 services, mainly larger Asia Europe services. The *11-services* instance is the size of a smaller liner shipping company operating 11 services, 237 port visits and 8,435 demands. In the two *Cluster* instances we have selected a set of ports in the same region such as the Baltic Sea or the Arabian Gulf and opened visits to these ports on many different services. In this case some but not all of the port visits on each service is opened. The open demand is the demand which goes through one of these opened port visits. The case *One-operator* contains 84 services and corresponds to a network operated by a major liner shipping company. For the instance *Whole-network* we have opened all port visits and all the services on the network can now be changed. Note that the number of demands is the same for the two cases *One-operator* and *Whole-network*.

We investigate the savings in fuel consumption as a function of the transit time limit. Three different cases are constructed: In the first case all transit times for customers have to be the same as in the current schedule. In the second case, 48 hours has been added to the limit on transit times. This means that many goods are delivered up to two days later than originally planned, which may be acceptable for many customers. Finally, the third case allows all current customer transit times to be violated. Since the overall duration of round trip is retained, container transit times will still be reasonable.

We have assumed that the bunker price is \$600 per ton and have a penalty for changing a port visit time to \$1000 (per week) resulting in a penalty of \$52000 per year. The penalty for changing a port visit was settled in collaboration with the liner shipping company but can be adjusted for each port if desired.

6. Results

The test cases presented in Table 3 have been solved using CPLEX 12.5 on a Linux computer with a 64 bit Intel Xeon 2.67 GHz CPU. The results from the tests are shown in Table 4. The instance name is provided in the first column. The second column shows the number of hours by which the transit times are allowed to change with respect to the current transit time. We test transit time limits as given by the current network, as well as limits extended with 48 hours and unlimited transit times. An extended time limit of 48 hours corresonds to at most 1–2 days longer transportation times which often will be accepted by customers. Unlimited transit times are used to illustrate the potential savings of neglecting time limits. Notice that since the overall duration of a round trip is constant, transit times still will not grow much since a slower speed on one leg means that another leg needs to be traversed at a higher speed.

Note that it is possible to set individual transit time requirements on each demand depending on their priority. However here it is chosen to set them to the same extended time for easy visual validation of the results and due to lack of data involving demand priorities. Since the current port visit times is a solution to the problem where no port visit has been changed we provided this solution to the solver for warm starting.

Columns 3–6 in Table 4 reports the result of the tests. Column three shows how many port visit times have changed. Here it is important to remember that there is a significant penalty introduced for changing the port visit time. The fourth column shows the reduction in fuel consumption achieved from optimizing. This improvement is the cost C_l on the legs l where the duration may be changed due to open port visit time compared to the cost of these legs on the original network. The cost of the legs which has not been selected for optimization is disregarded as no savings can occur on them. The fifth column shows the running time in seconds and the last column shows the gap between the lower bound and the best found feasible solution. Note that this gap also includes the penalty costs.

The cases show promising savings and the running times for the two cluster instances, and the one and two string instances are less than two minutes with most of them around 1 to 5 seconds. The instance *11-services* does not reach optimum in the 10 minutes provided. However it is very close to optimum despite the fact that the instance is very large with 237 open port visits and 8435 demands going through the open port calls.

If the original transit time limits are used, savings of up to 8% can be achieved. With slightly extended transit time limits of 48 hours, the savings are in the magnitude of 1-13% showing that major savings can be achieved without significantly changing the service level. If no transit time limits are present, the solutions can be slightly improved, but generally the quality of solution is similar to the solutions found by extending the transit time limit by 48 hours.

The last cases containing the entire network of one major operator and the whole network of many operators are not solved to optimality within the 10 minutes. However a whole network containing different operators is not a realistic case and it is also unlikely that a major operator will try to reschedule the port visit on their entire network at once.

Another interesting observation is that a substantial saving can be obtained by increasing the transit time limit by 48 hours, but incrasing the transit times further does not significantly improve the savings. The saving achieved on the two cluster instances are very different. This could be due to a difference in how well the bunker consumption has been optimized in the manual original planning for each of the clusters. However it could also be due to the fact that we only consider standard bunker and do not consider low sulphur bunker in the problem and the Baltic is a region where low sulphur bunker must be used.

Instance	added transit times (hours)	port calls changed	cost reduction	time (s)	gap
Single-service	0	5	1.12%	0	optimal
Single-service	48	13	3.99%	1	optimal
Single-service	∞	13	4.54%	4	optimal
Two-services	0	8	0.65%	0	optimal
Two-services	48	24	4.62%	1	optimal
Two-services	∞	23	5.63%	63	optimal
11-services	0	34	0.74%	6	optimal
11-services	48	177	7.10%	600	0.08%
11-services	∞	190	8.98%	600	0.09%
Cluster-1	0	26	8.67%	0	optimal
Cluster-1	48	47	13.39%	3	optimal
Cluster-1	∞	38	13.48%	0	optimal
Cluster-2	0	0	0.00%	0	optimal
Cluster-2	48	21	1.77%	1	optimal
Cluster-2	∞	19	2.29%	0	optimal
One-operator	0	112	0.82%	175	optimal
One-operator	48	170	1.28%	600	6.29%
One-operator	∞	589	7.93%	600	1.99%
Whole-network	0	250	1.90%	600	0.14%
Whole-network	48	245	6.90%	600	1.20%
Whole-network	∞	790	9.16%	600	0.35%

Table 4: Test results with the different instances described in Table 3. The time limit is 10 minutes.

7. Conclusion

We have presented a model which can find the optimal schedule for an existing network. The test results show that it is possible to reduce bunker consumption significantly for real-life services simply by rescheduling the port visit times while only introducing minor reductions in the service level. A large liner shipping company such as Maersk Line transports nearly 9 million FFE (Forty Foot Equivalent Unit) per year, using around 1000 kg bunker (2013) per FFE, so a reduction of just a few percent will give substantial savings measured in absolute numbers.

The cases where all port visits on the entire network are allowed to be rescheduled may provide savings of that size. The interesting results from the tests is that significant savings already appear when allowing the transit time to increase with two days (48 hours). In a competitive environment such savings are important. We show that if only parts of the network are rescheduled the problem can be solved fast. However, if the whole network is rescheduled and limits are applied on all the transit times then the solver was not able to close the gap to the lower bound but significant reductions in cost is still achieved. Since there are more than 20,000 demands in a real-life network and each transhipment in the presented model introduces new integer variables the demands are the primary cause for the increased running time.

Issues such as Suez Canal meet up times can be handled by locking the port visit. However to be sure to achieve an optimal solution a future implementation would be to restrict the rescheduling to ensure meet up times when dealing with Canals. Another improvement for the model could be the handling of Emission Control Areas where another more expensive fuel type must be used. Incorporating this requires information of where the Emission Control Area is entered.

We have shown that the model can solve realistic problems of a reasonable size and achieve good solutions to large problems and improve the current solutions.

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